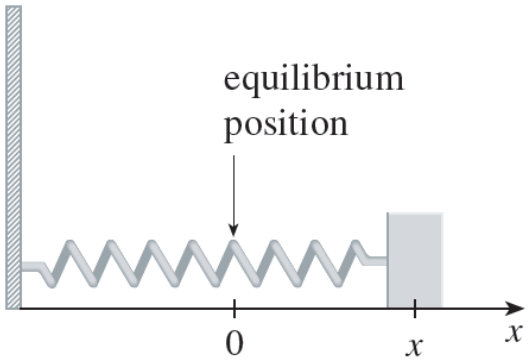
Exercise 35

A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is $x(t) = 8\sin t$, where t is in seconds and x in centimeters?

(a) Find the velocity and acceleration at time t.

(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at that time?



Solution

The equation of motion gives the position of the mass at every point in time.

$$x(t) = 8\sin t$$

Differentiate it with respect to t to get velocity.

$$v(t) = x'(t) = \frac{d}{dt}(8\sin t) = 8\cos t$$

Differentiate the velocity to get acceleration.

$$a(t) = x''(t) = \frac{d}{dt}(8\cos t) = -8\sin t$$

Evaluate each of these functions at $t = 2\pi/3$.

$$x\left(\frac{2\pi}{3}\right) = 8\sin\frac{2\pi}{3} = 4\sqrt{3}\text{ cm}$$

$$v\left(\frac{2\pi}{3}\right) = 8\cos\frac{2\pi}{3} = -4\ \frac{\text{cm}}{\text{s}}$$

$$a\left(\frac{2\pi}{3}\right) = -8\sin\frac{2\pi}{3} = -4\sqrt{3}\ \frac{\text{cm}}{\text{s}^2}$$

These results indicate that at $t = 2\pi/3$ the mass is at $x = 4\sqrt{3}$ cm, moving to the left with a speed of 4 cm/s, and gaining speed to the left with an acceleration of $4\sqrt{3}$ cm/s^2.